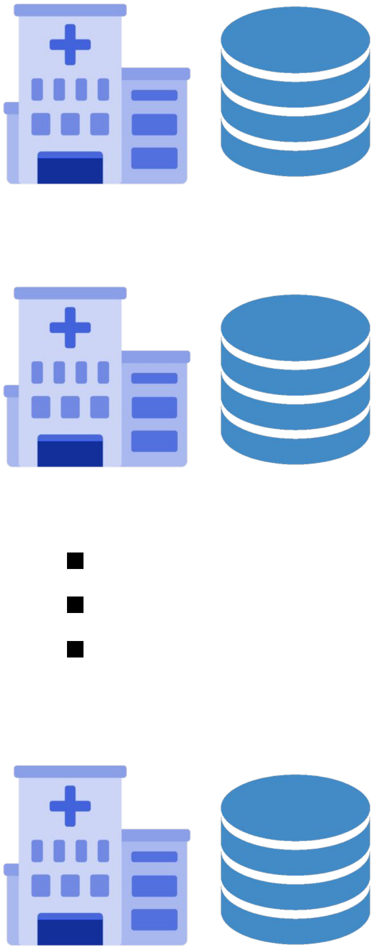
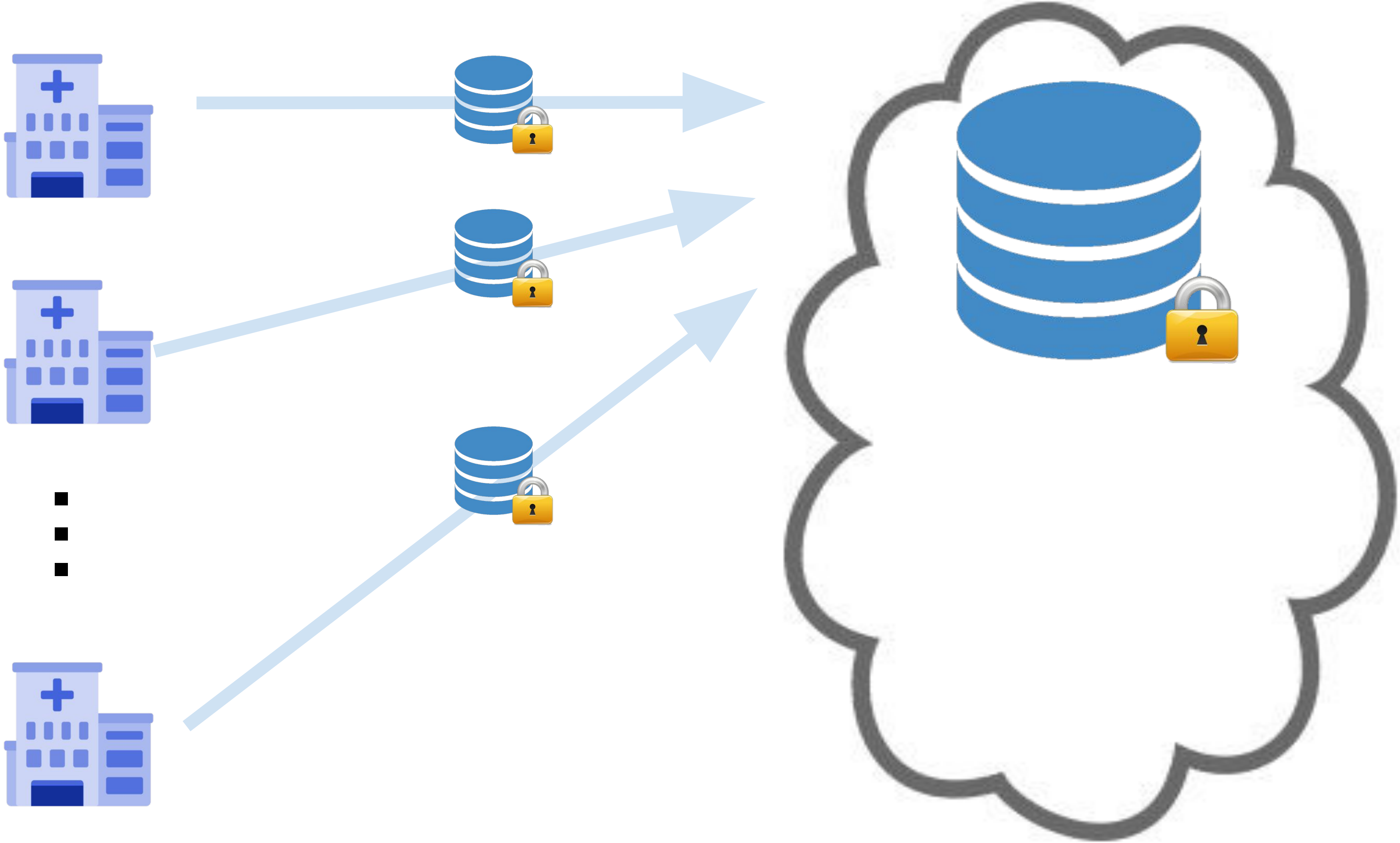
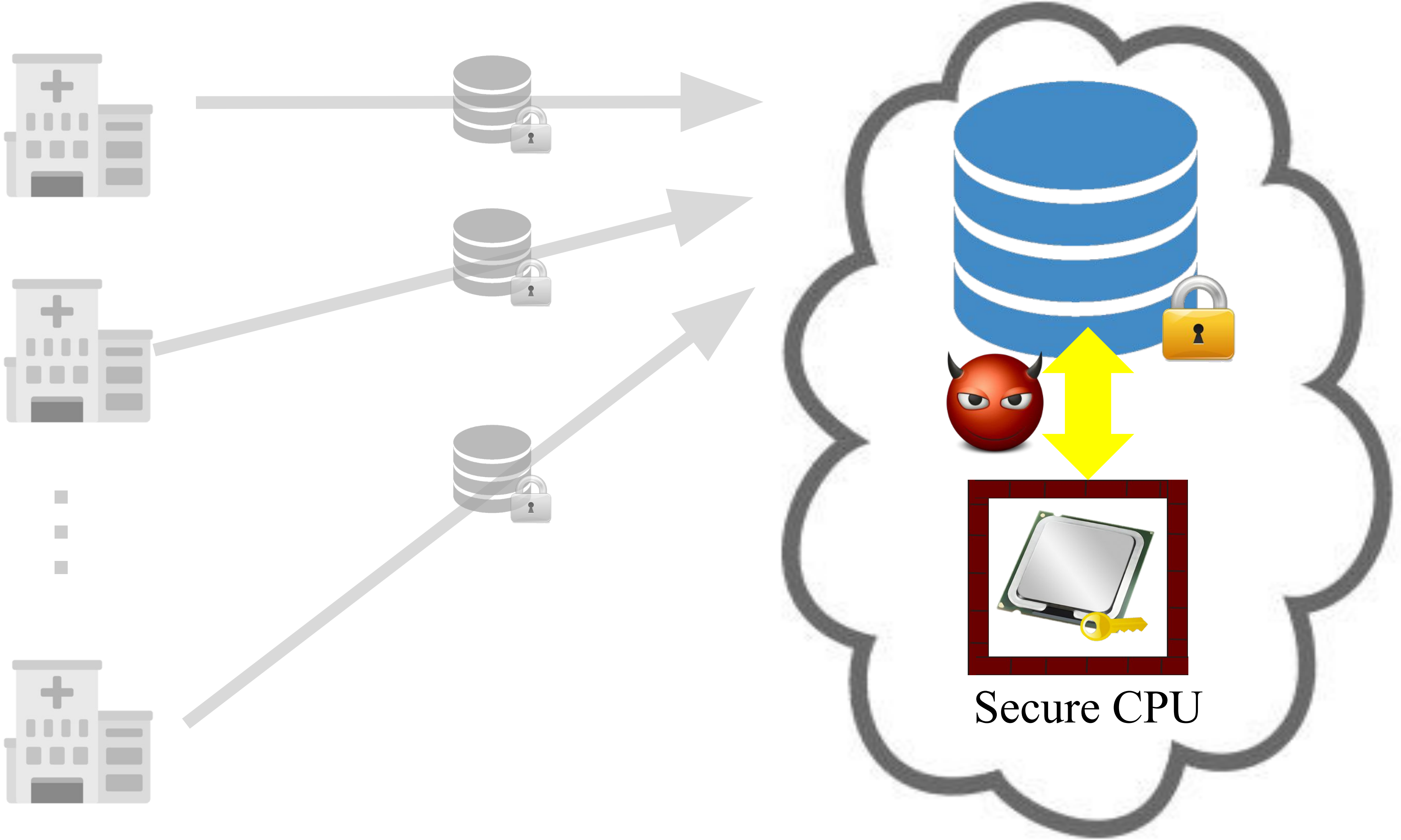
Foundations of Differentially Oblivious Algorithms

Elaine Shi

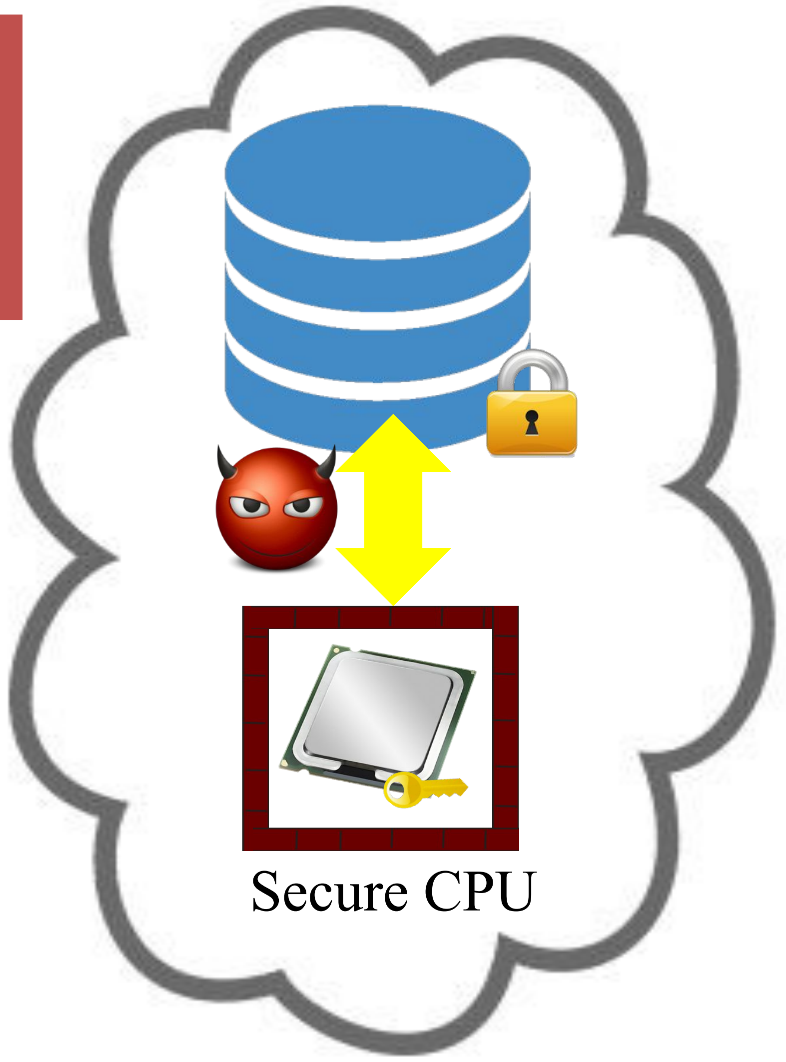
Based on [CCMS'18] and [LSX'18]





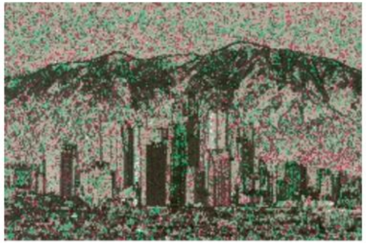

Secure CPU

Access patterns to even encrypted data leak sensitive information.



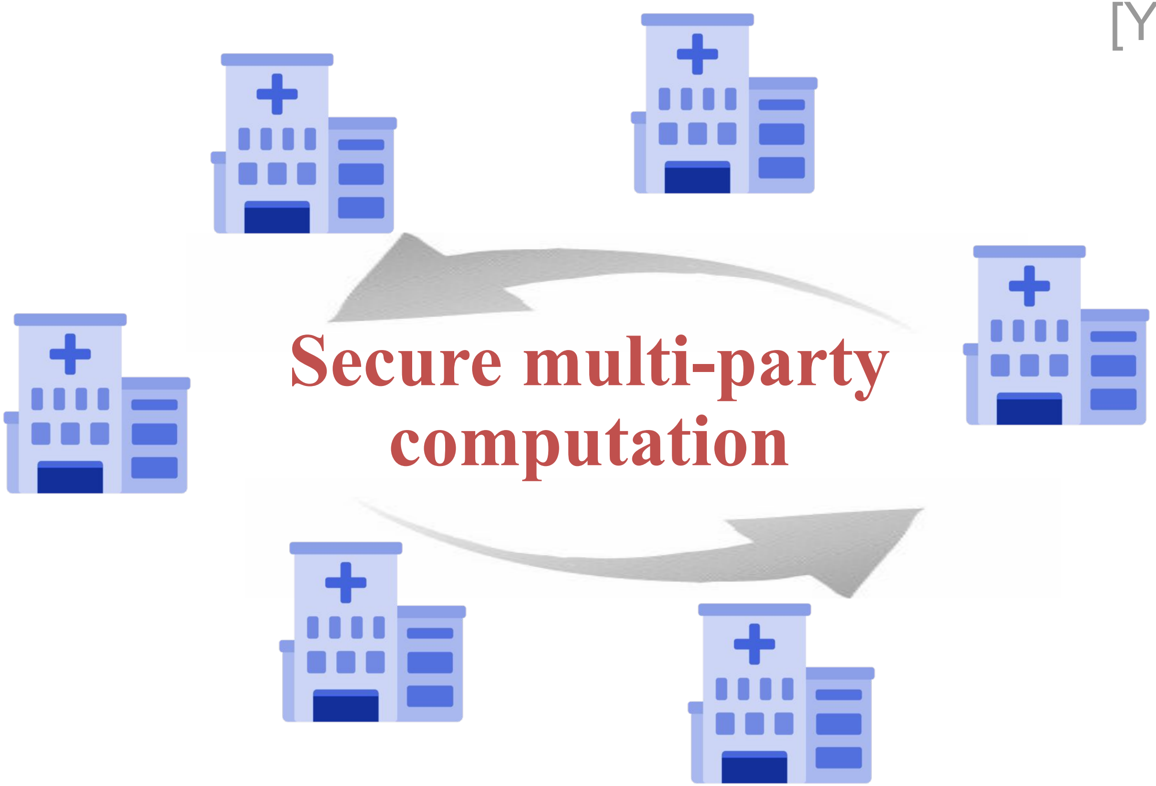
Access Pattern Attack: Computing on JPEG Image

Original Recovered



Controlled-Channel Attacks [XCP’15]

[Yao'82, GMW'87]



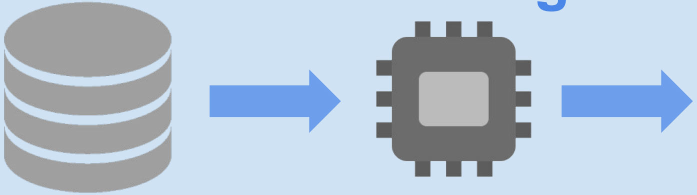
Access Pattern Leakage in MPC

Oblivious RAM

An algorithmic approach that provably obfuscates access patterns


Oblivious RAM

Oblivious algorithm



Real-world addresses

≈

Simulator → Simulated addresses

Oblivious RAM

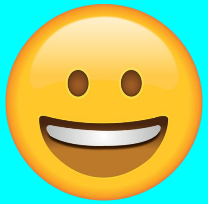
"Encrypting the access patterns"

Oblivious RAM

"Encrypting the access patterns"

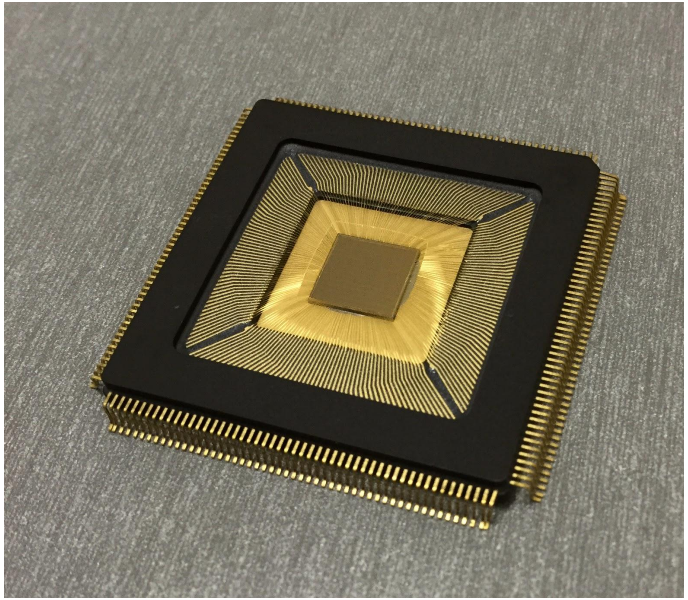
- Permute data in memory
- Shuffle data upon accesses

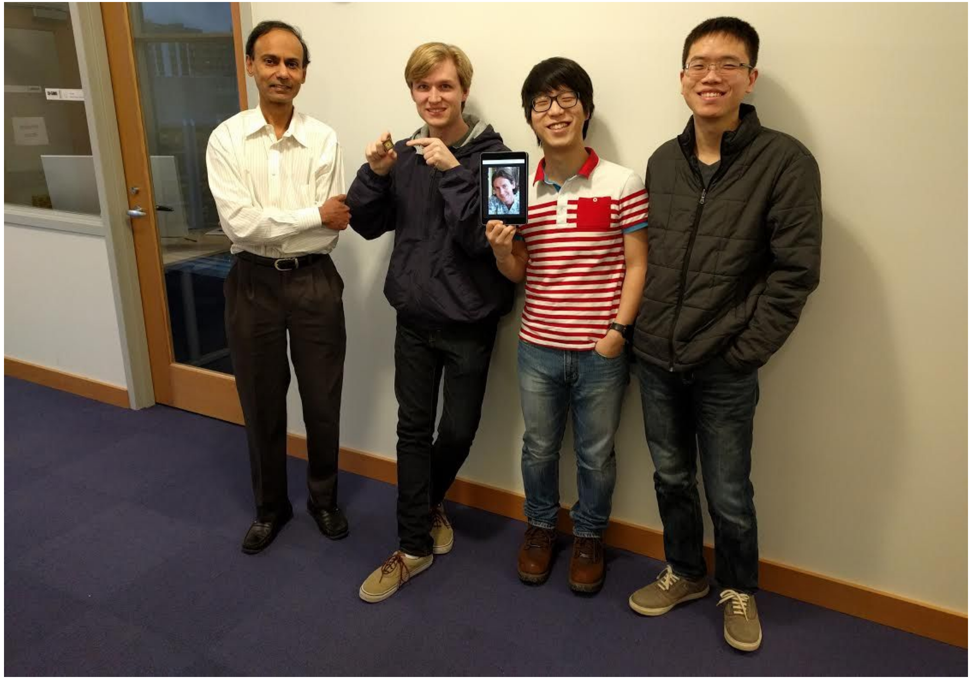
ORAM State of the Art



Any program can be made oblivious with $O(\log N)$ to $O(\log^2 N)$ overhead

[Optoroma, Circuit ORAM, ...]





ORAM State of the Art

Any program can be made oblivious with $O(\log N)$ to $O(\log^2 N)$ overhead

$\Omega(\log N)$ is necessary

[GO'96, LN'18]

ORAM State of the Art

Any program can be made oblivious with $O(\log N)$ to $O(\log^2 N)$ overhead

Implicit assumption:

Runtime is fixed and known

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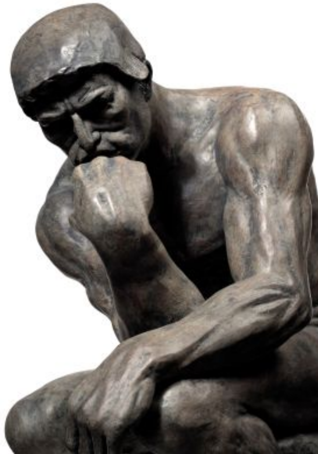
Runtime is fixed and known

- Must pad to worst-case runtime
- Can incur even linear overhead

Relax the obliviousness notion?

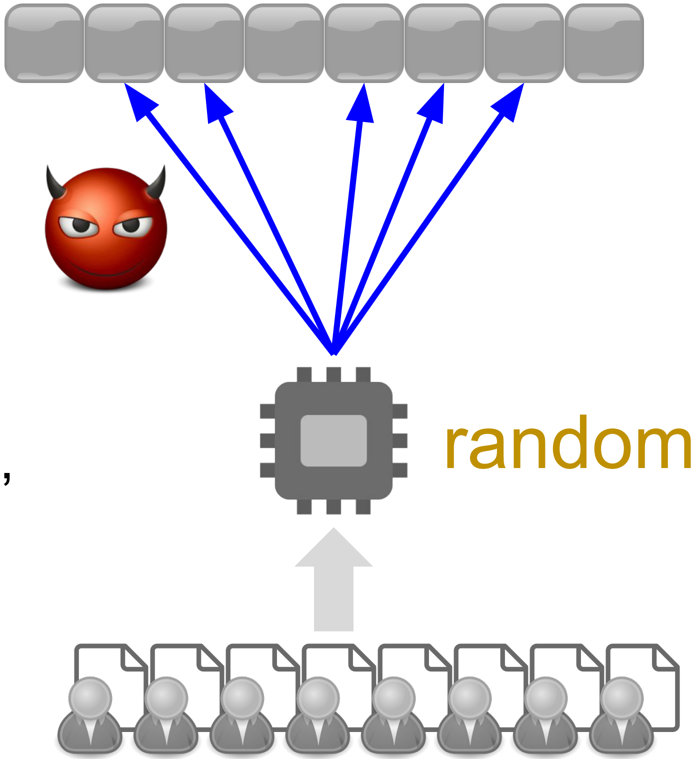
- Still provide meaningful privacy
- Significantly improve efficiency



Differential Obliviousness

Inspired by differential privacy [Dwork et al. 05]

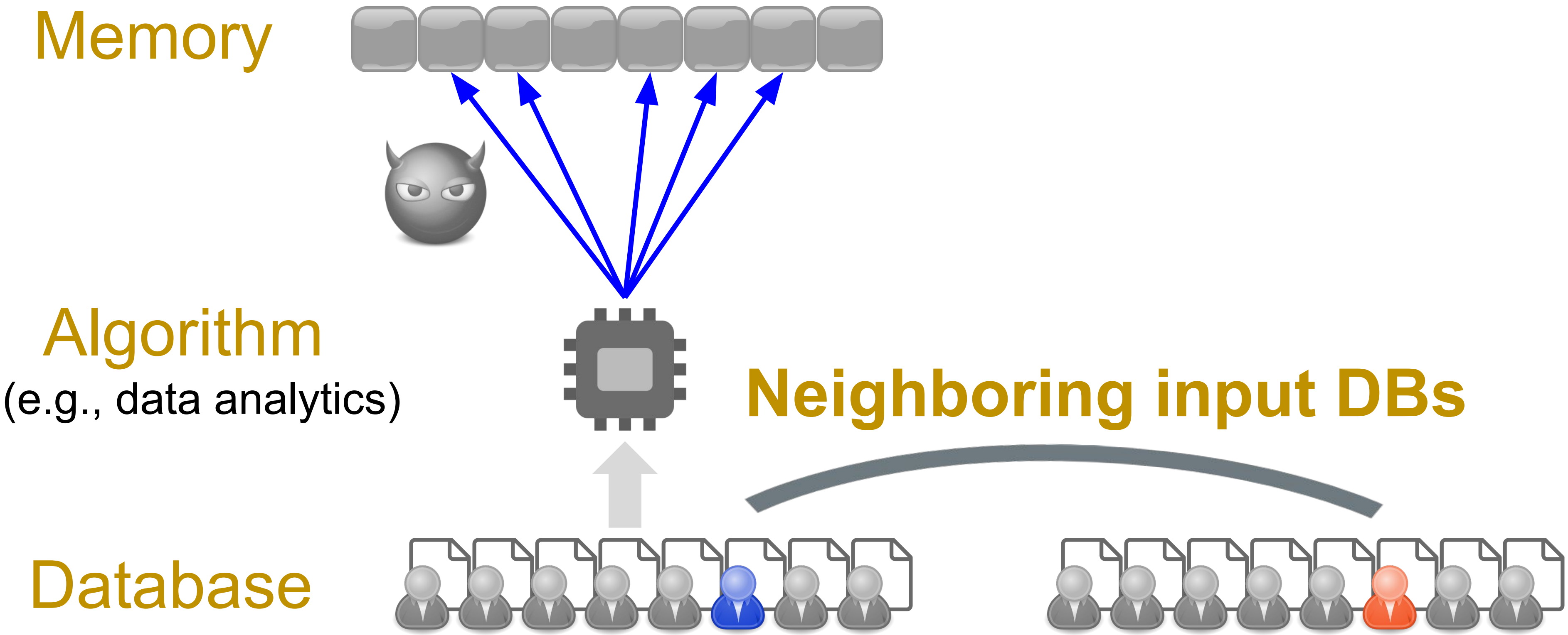
Memory

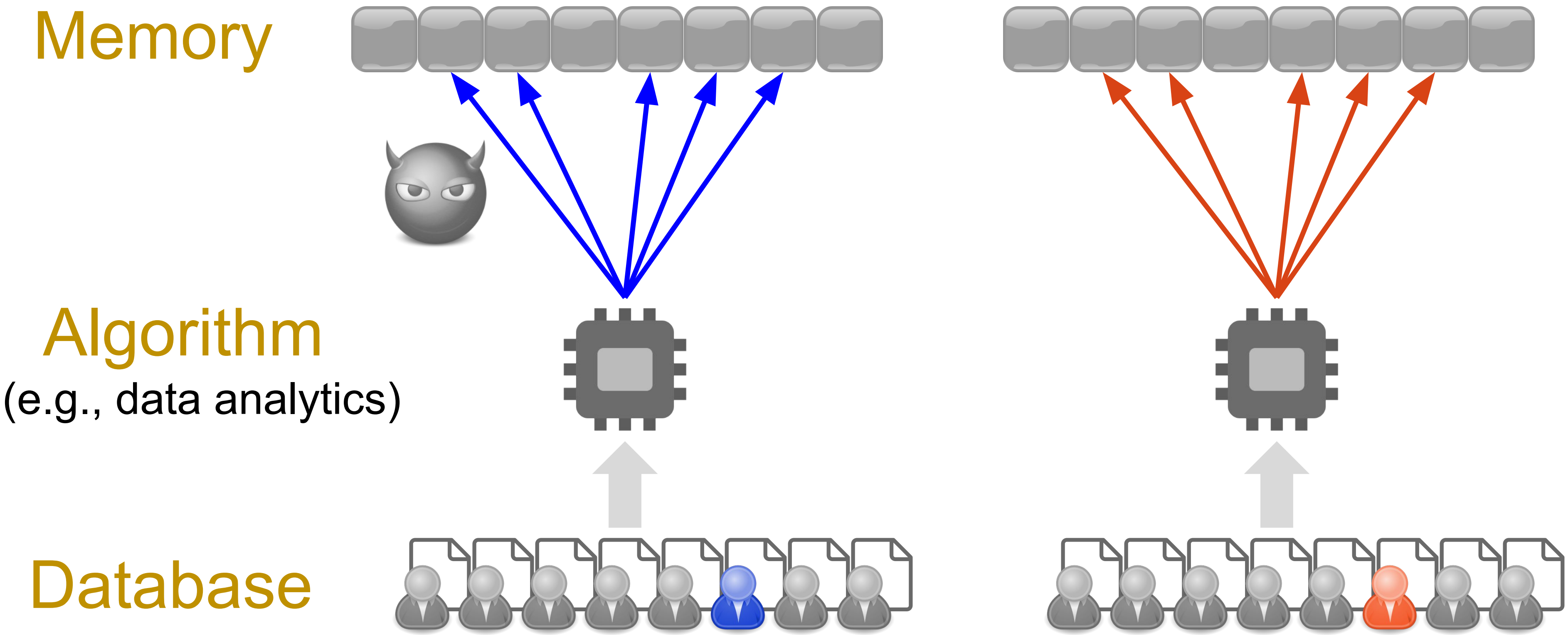


Algorithm
(e.g., compaction, sorting)

randomized

Database


Memory
Algorithm
(e.g., data analytics)
Neighboring input DBs
Database


Memory
Algorithm
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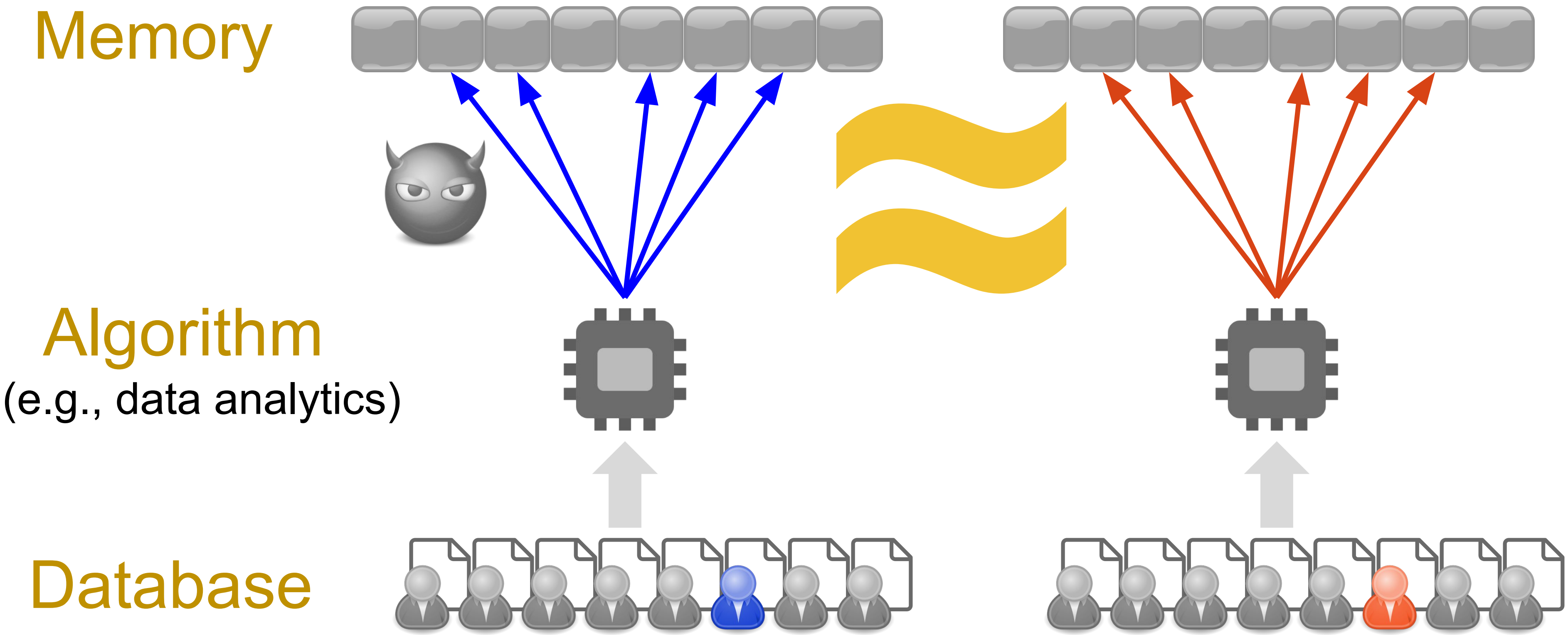
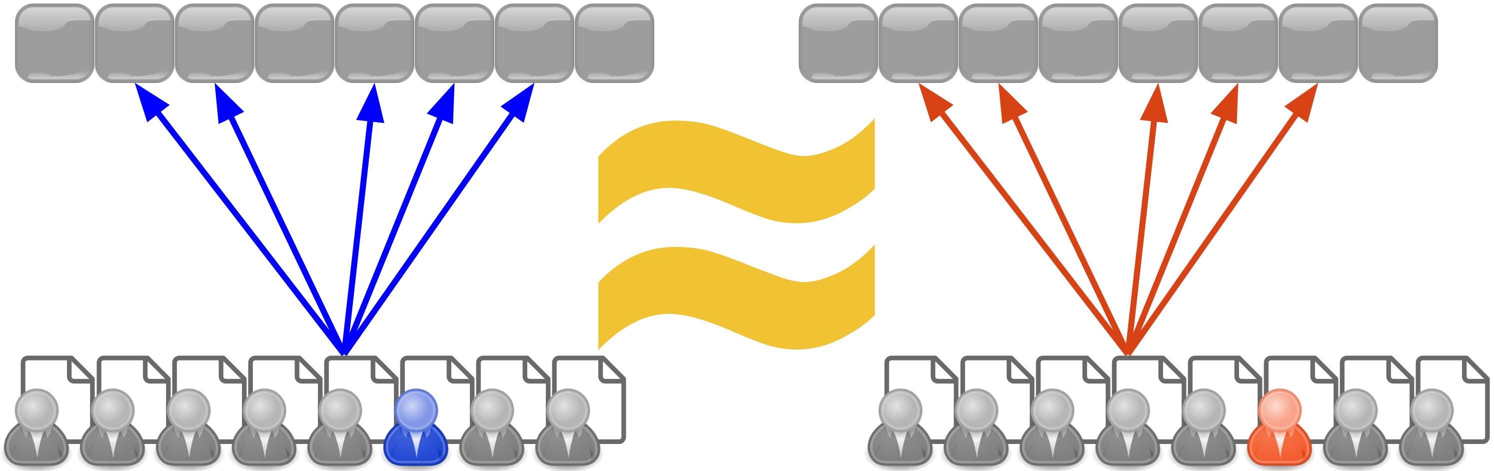
Database

Access patterns on neighboring DBs must be close



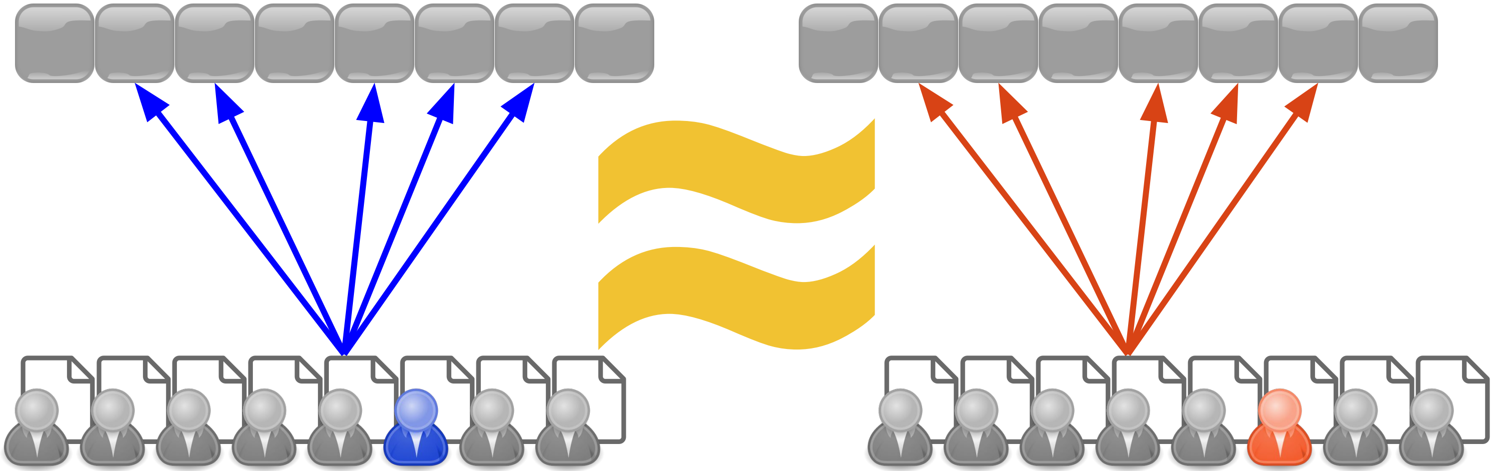
Access patterns on **neighboring** DBs must be **close**

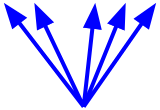
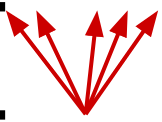


$$\Pr[\ \in S] \le e^{\epsilon} \Pr[\ \in S] + \delta$$

This must hold for any S

(ϵ, δ)-Differential Obliviousness



Pr[ $\in$ S] $\leq e^{\epsilon}$ Pr[ $\in$ S] + δ

This must hold for any S

(ϵ, δ)-Differential Obliviousness

1. What is being relaxed?
2. Still provide meaningful privacy?
3. Overcome obliviousness barriers?

(ϵ, δ)-Differential Obliviousness

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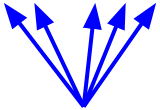
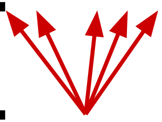
What is being relaxed?

Closeness needs to hold only for neighboring DBs

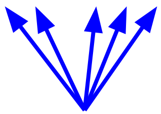
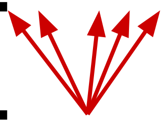
Pr[ $\in$ S] $\leq e^{\epsilon}$ Pr[ $\in$ S] + δ

This must hold for any S

What is being relaxed?

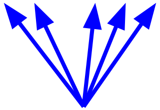
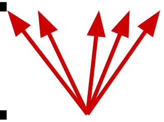
Closeness needs to hold only for neighboring DBs

Allow multiplicative, non-negl. loss

Pr[ $\in S] \leq e^{\epsilon}$ Pr[ $\in S] + \delta$

This must hold for any S

Does not require padding
to worst-case runtime

Pr[ $\in S] \leq e^{\epsilon}$ Pr[ $\in S] + \delta$

This must hold for any S

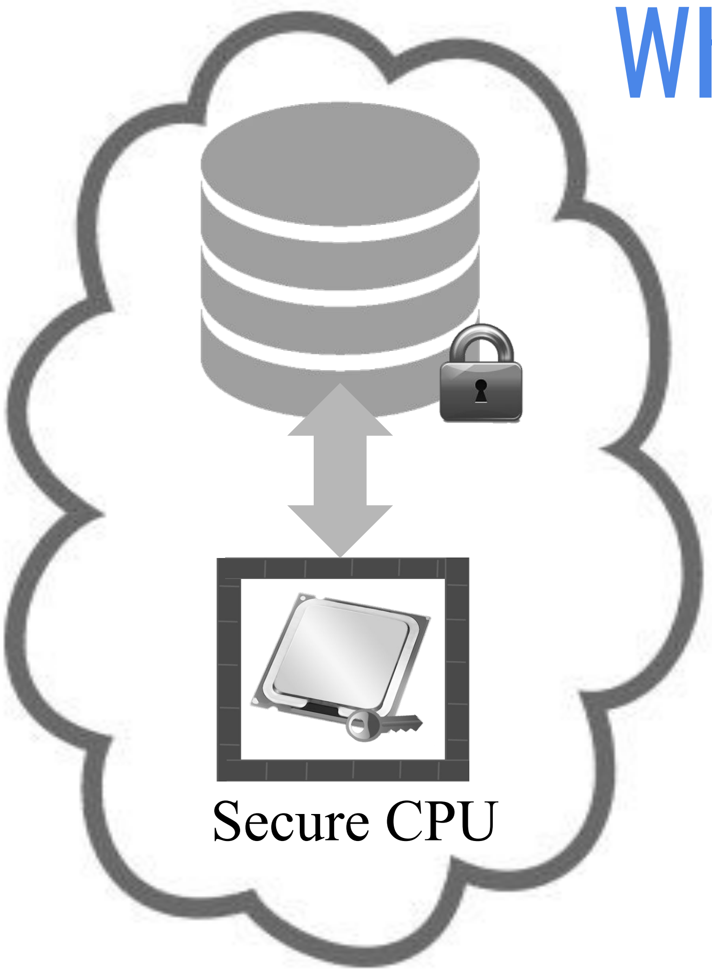
(ϵ, δ)-Differential Obliviousness

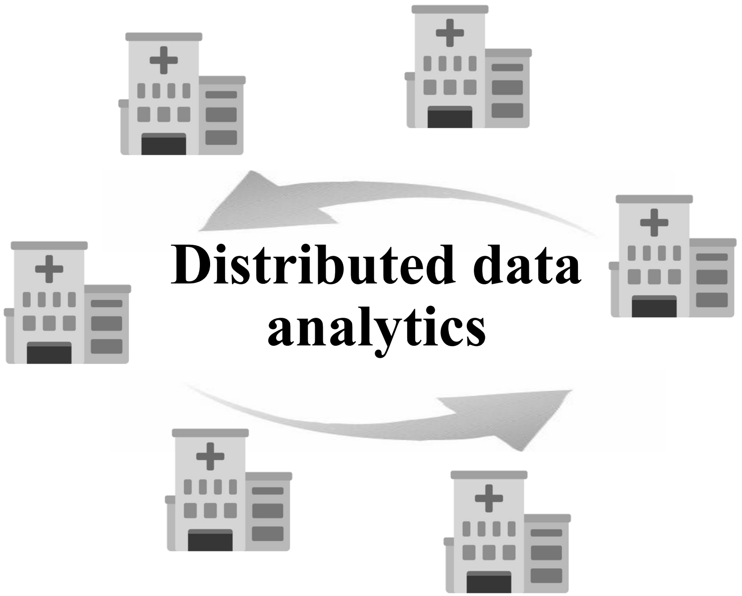
1. What is being relaxed?
2. **Still provide meaningful privacy?**
3. Overcome obliviousness barriers?

Bad idea if you are protecting your Bitcoin signing key!

2 Still provide meaningful privacy?

When does DO make sense?

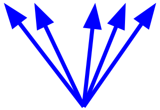
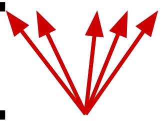




Typical parameters

Constant

Negl. in N

Pr[ $\in$ S] $\le e^{\epsilon}$ Pr[ $\in$ S] + δ

This must hold for any S

(ϵ, δ)-Differential Obliviousness

1. What is being relaxed?
2. Still provide meaningful privacy?
3. Overcome obliviousness barriers?

Stable Compaction

Obliviousness

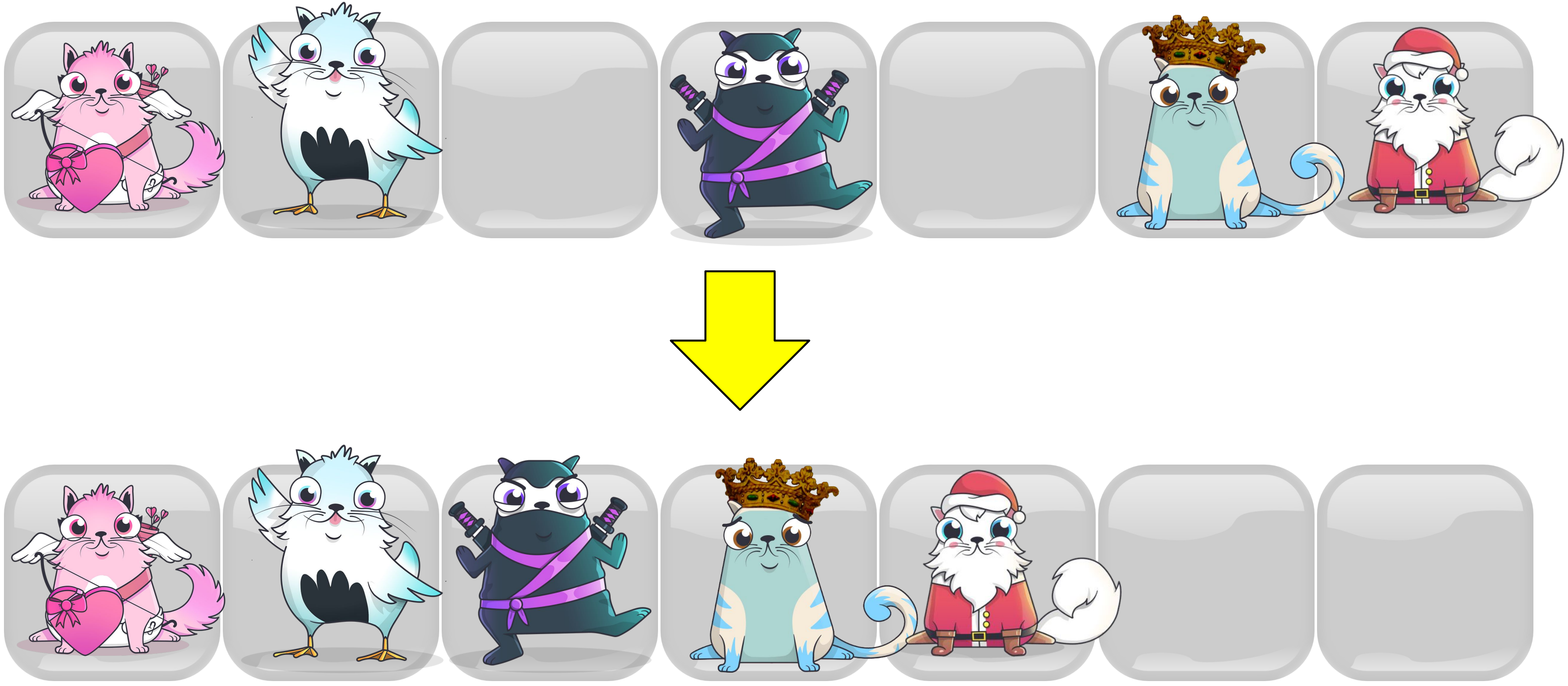
$\Omega(N \log N)$

necessary

Differential Obliviousness

$O(N \log \log N)$

Stable Compaction



Stable Compaction: Why do we care?

- Simple yet non-trivial
- Frequent algorithmic building block
- Warmup scheme in paper

Stable Compaction: insecure algorithm

Stable Compaction: insecure algorithm

Stable Compaction: insecure algorithm

Stable Compaction: insecure algorithm

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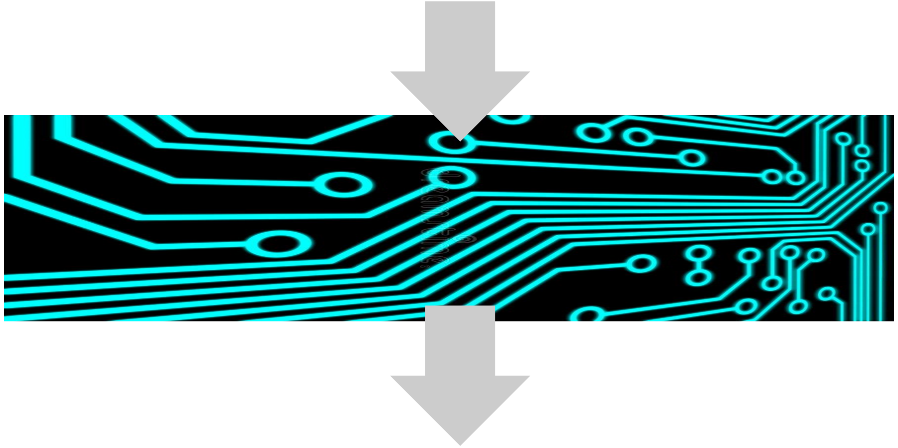
Completes in $O(N)$ time

Leaks exact progress

Stable Compaction: **oblivious** algorithm

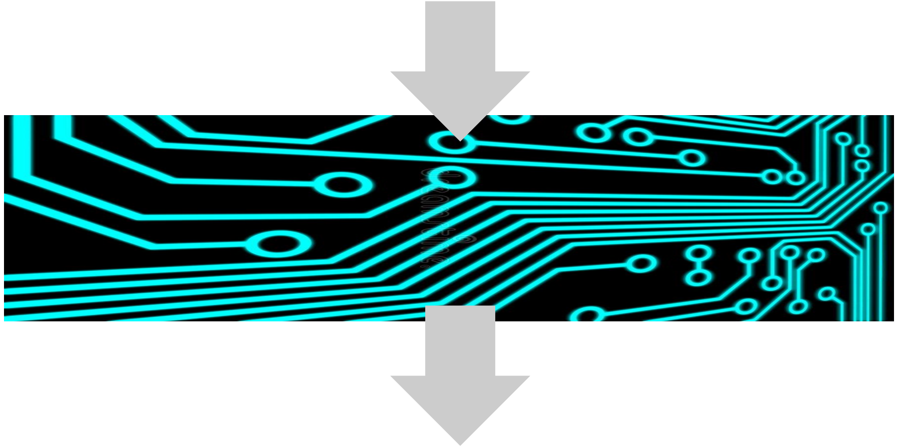
Stable Compaction: **oblivious** algorithm

Sorting network



Takes N log N time

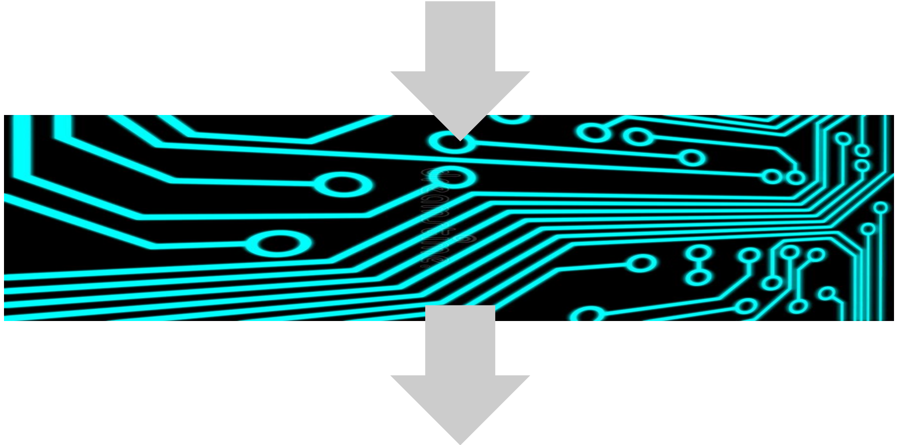
Sorting network



$N \log N$ time is necessary for obliviousness

Assumption: algorithm does not perform encoding on the kitties

Sorting network



Stable Compaction

Obliviousness

$\Omega(N \log N)$

necessary

Differential Obliviousness

$O(N \log \log N)$

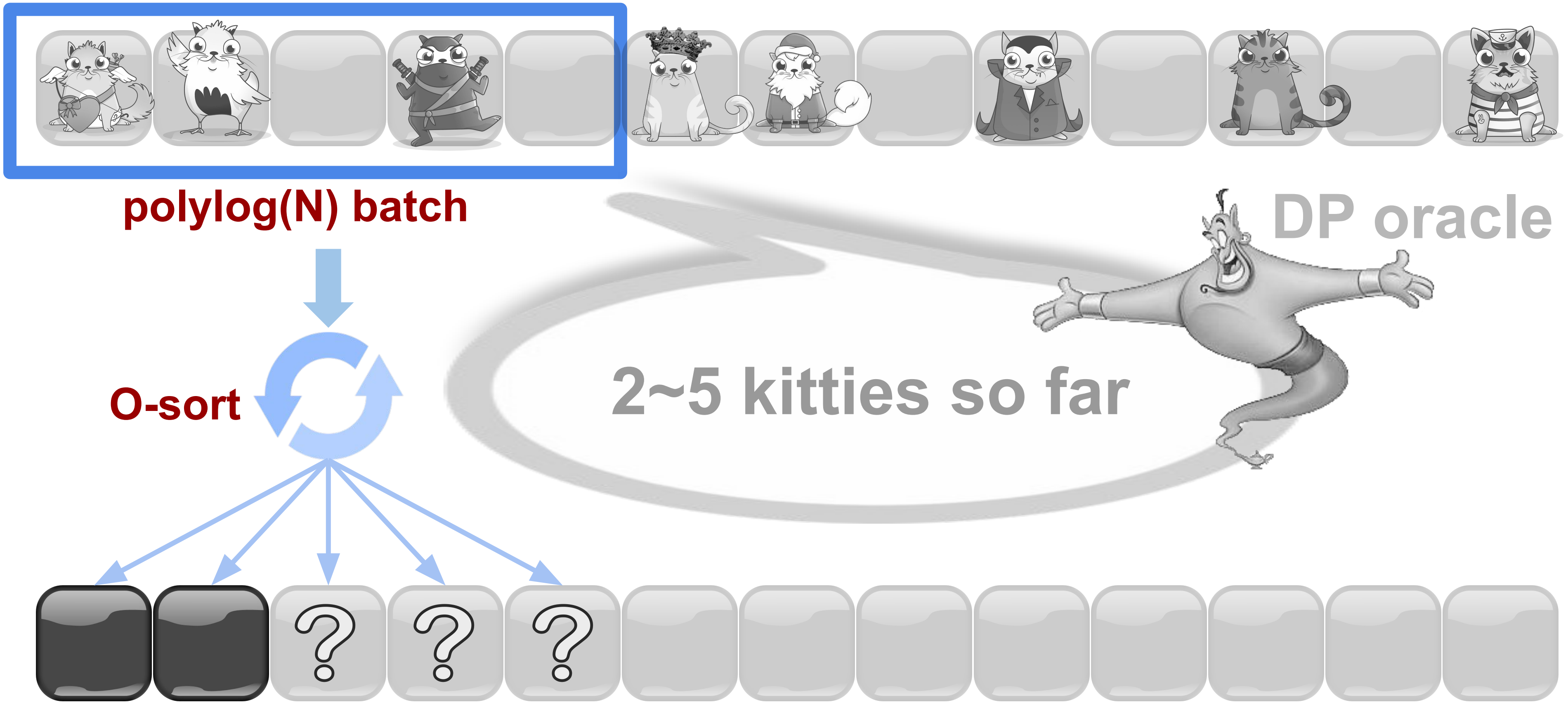
Obliviousness

Cannot leak progress

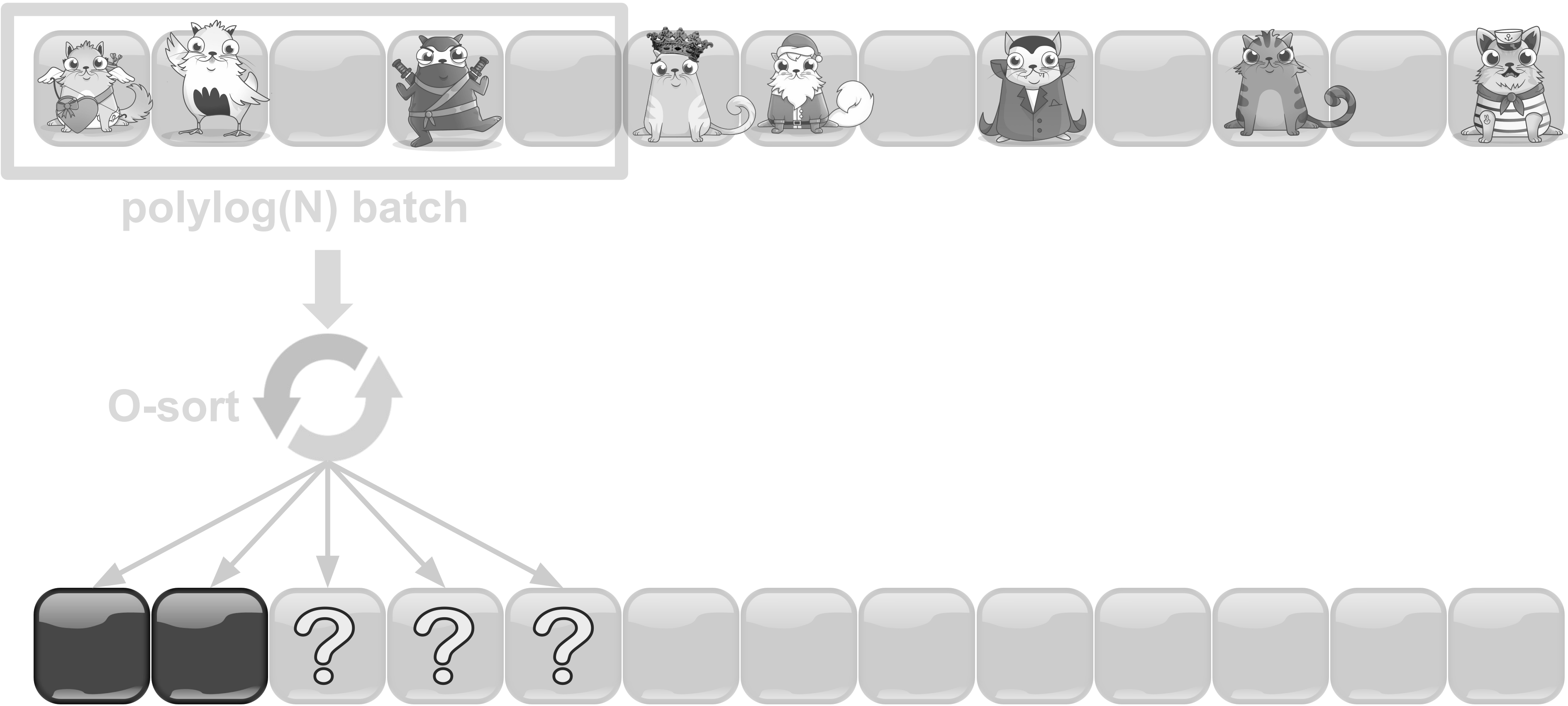
Differential Obliviousness

Leak rough notion of progress

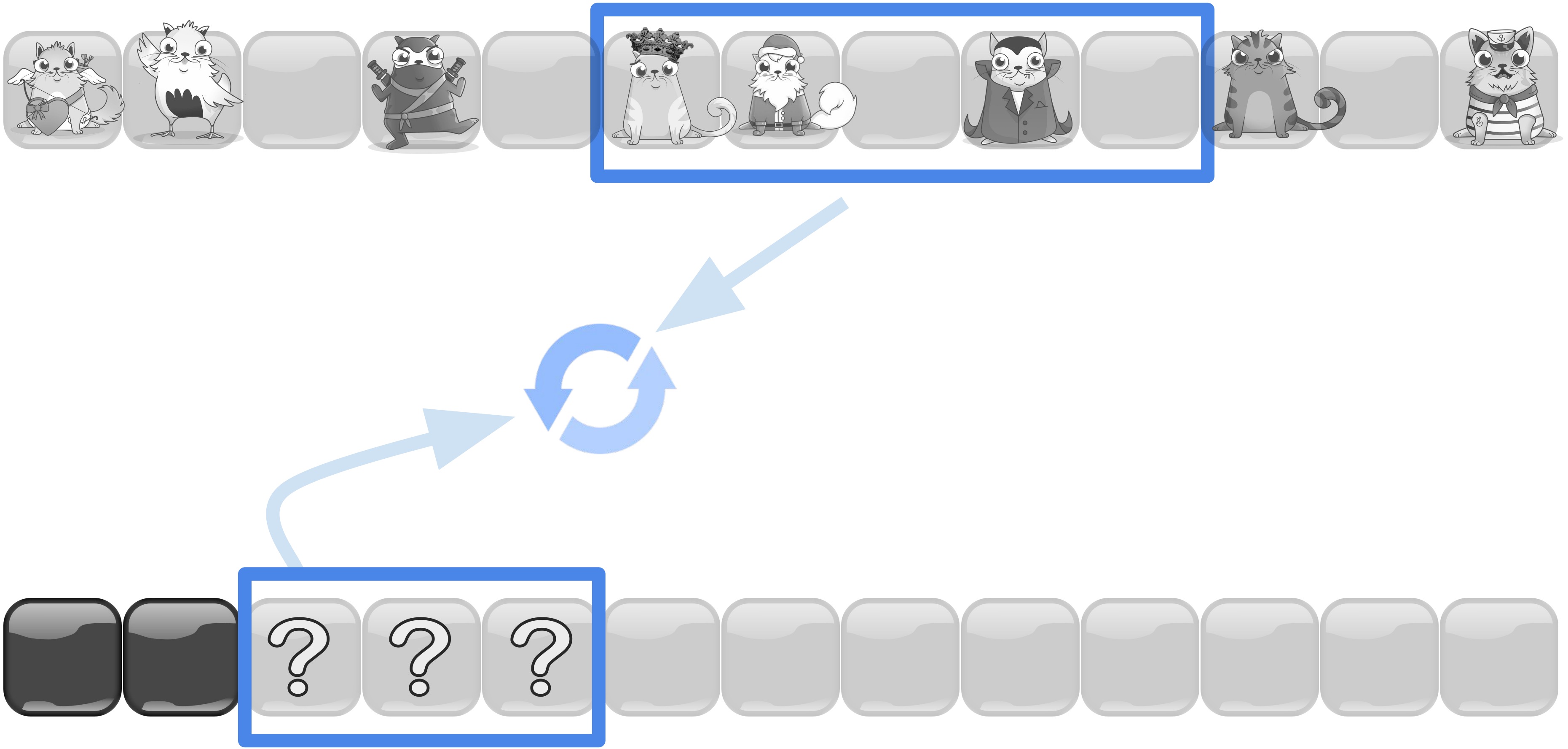

polylog(N) batch
DP oracle
2~5 kitties so far

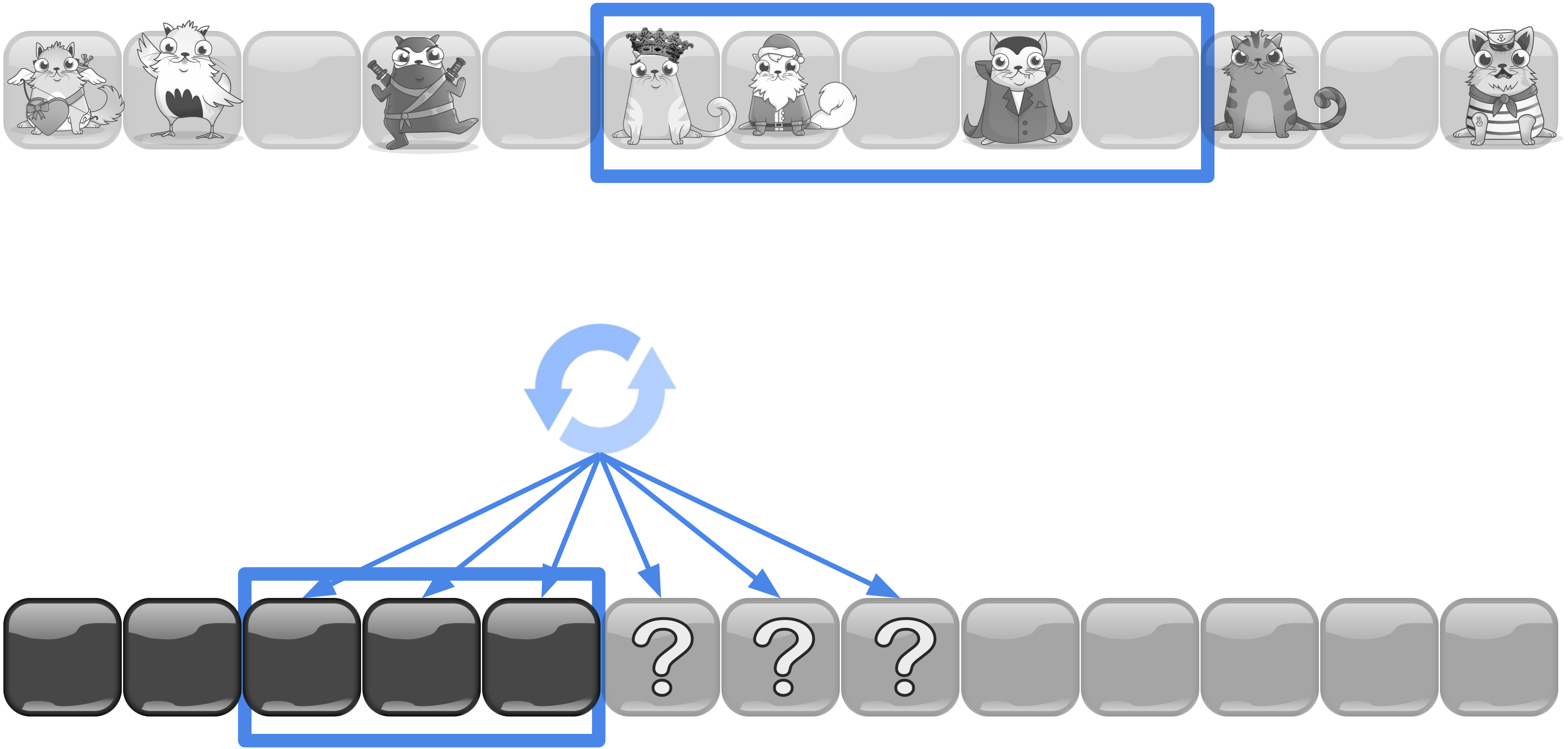


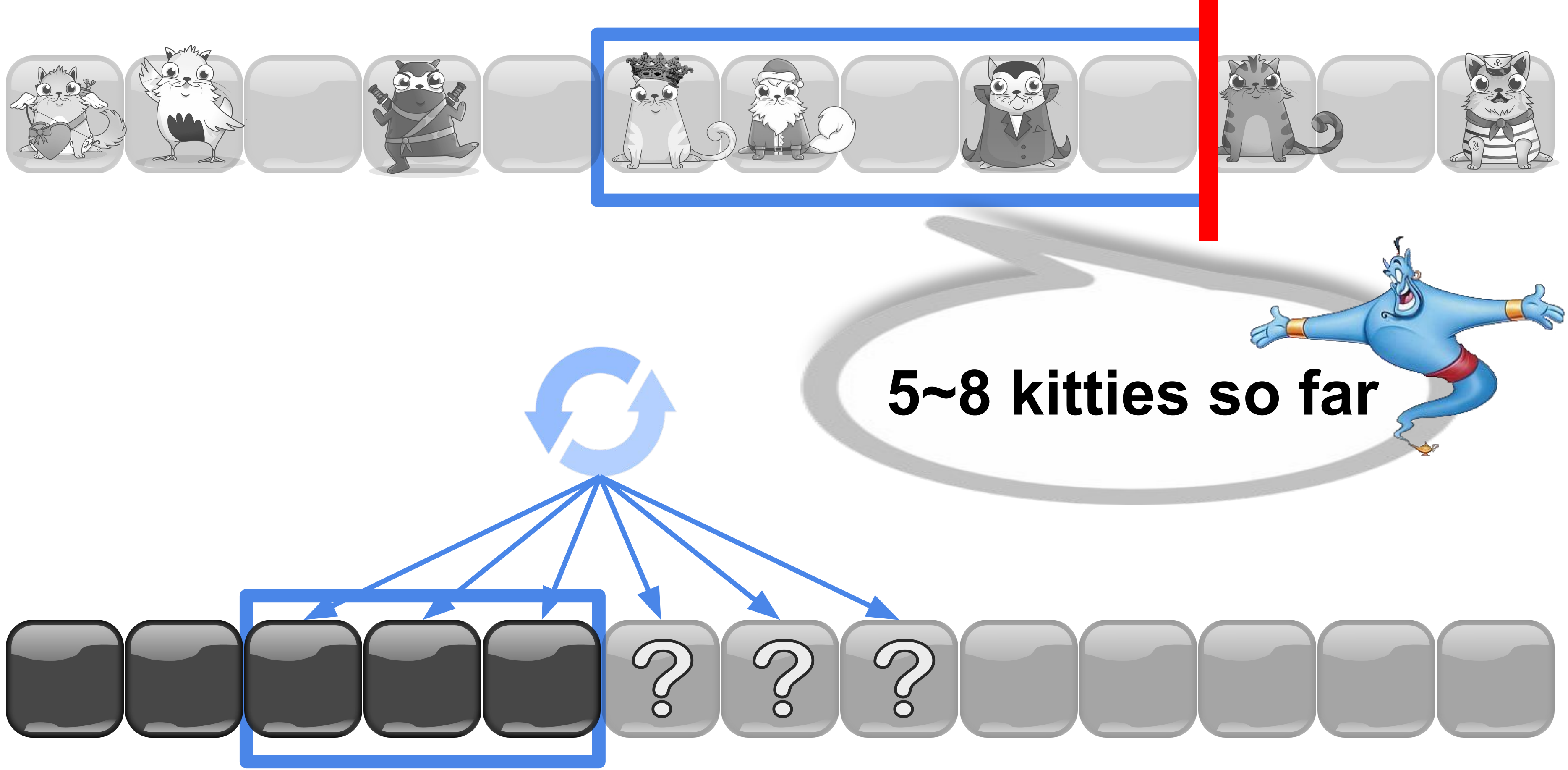
polylog(N) batch
DP oracle
O-sort
2~5 kitties so far
?
?
?

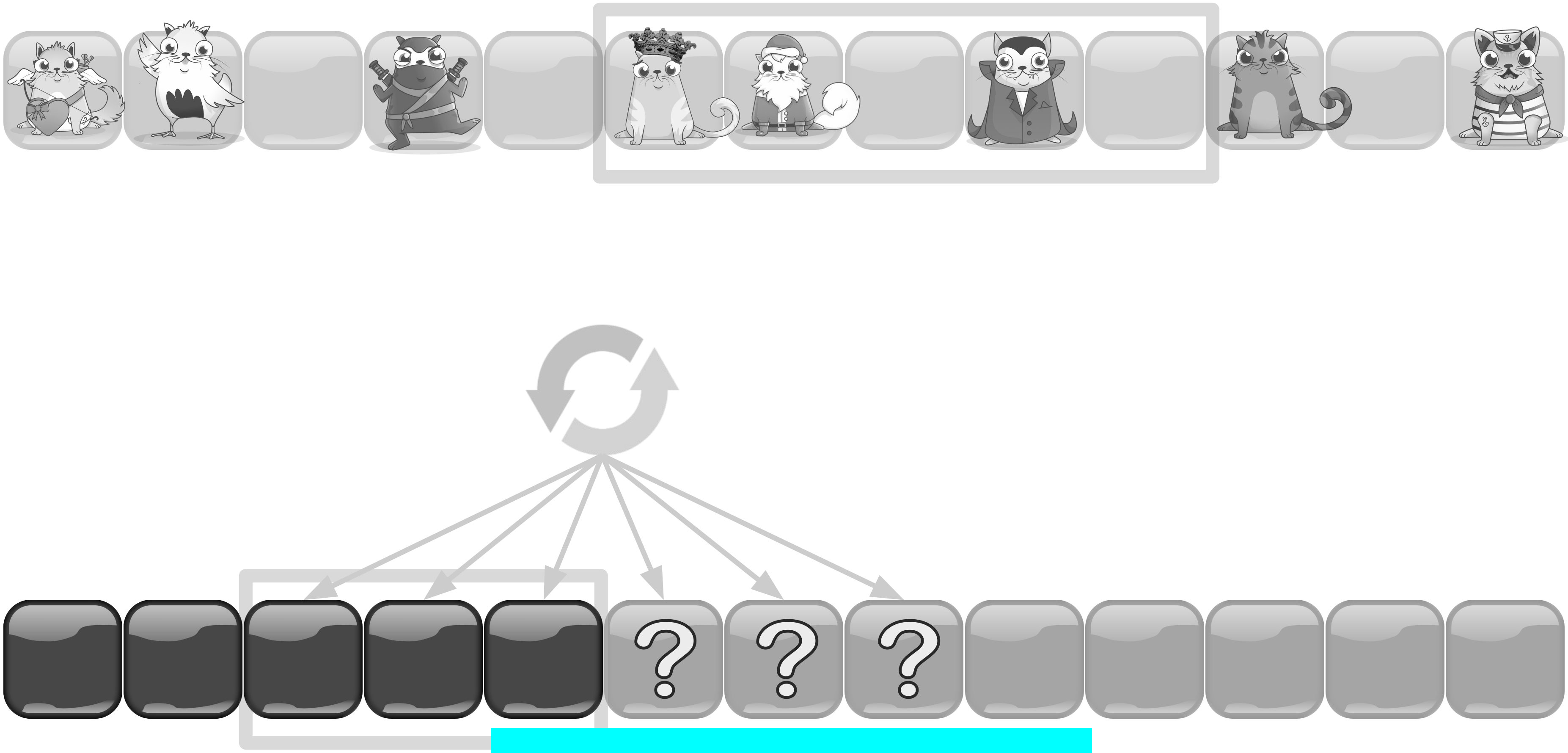


polylog(N) error,
DP estimate






5~8 kitties so far
?
?
?

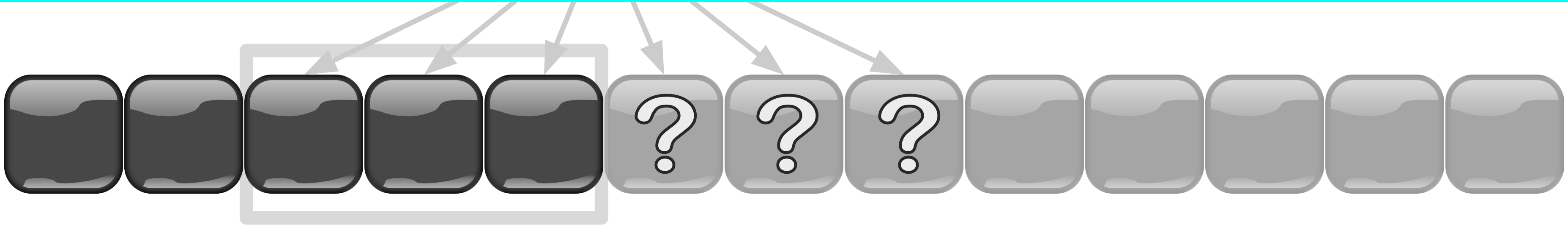


polylog(N) error,
DP estimate

Completes in

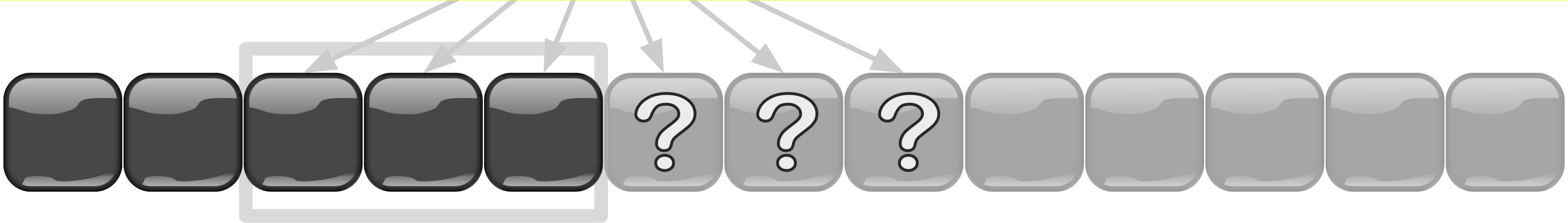
$O(N \log \log N)$

time



Need:

Oblivious and DP alg. that estimates all prefix sums, with polylog error



Naive algorithm:

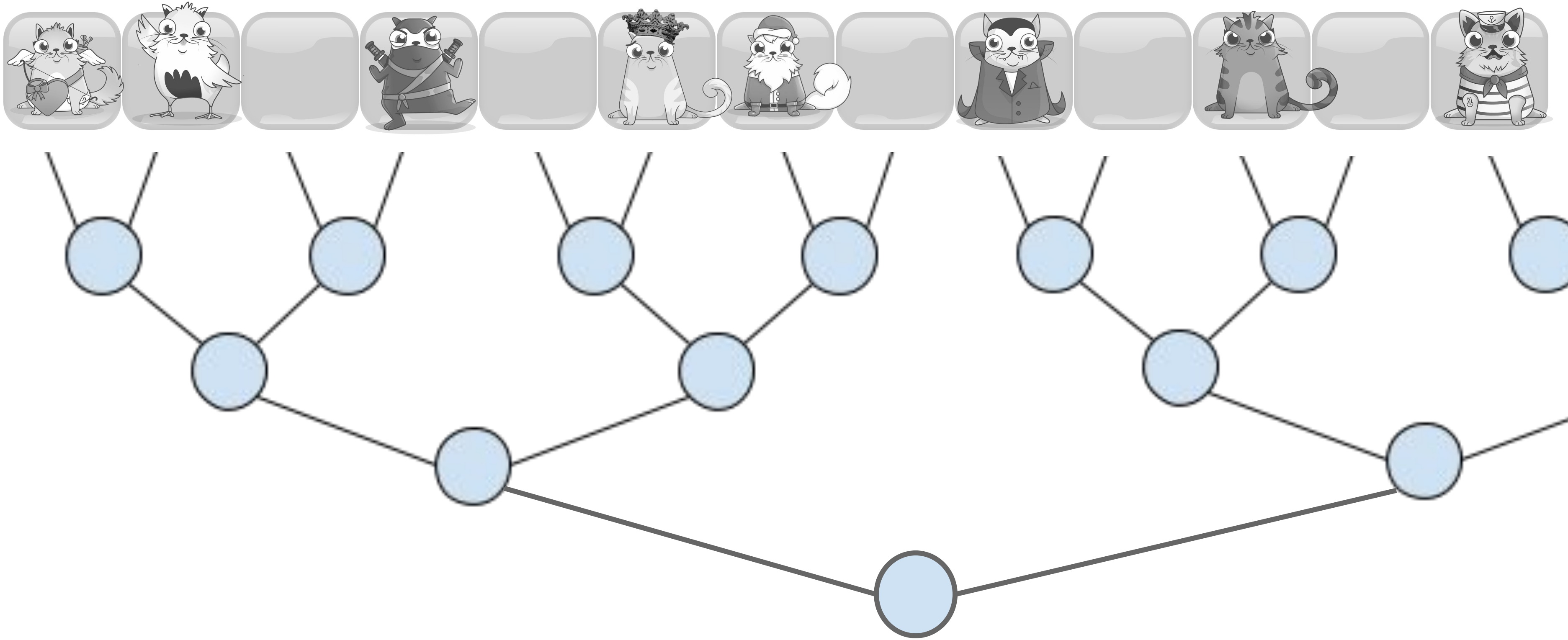
- Compute all N prefix sums
- Add independent noise to each

All prefix sums -- DP and Oblivious

Naive algorithm:

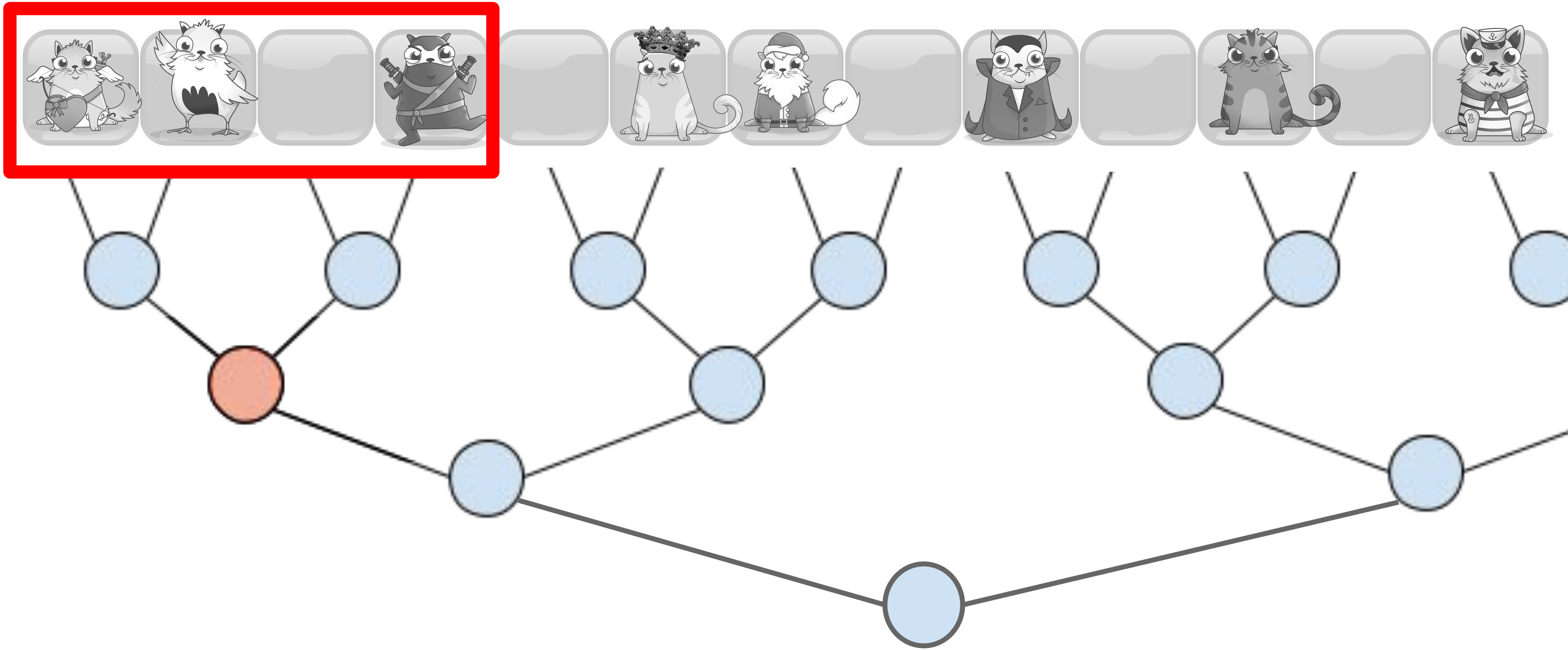
- Compute all N prefix sums
- Add independent noise to each

Incurs $\Theta(N)$ error

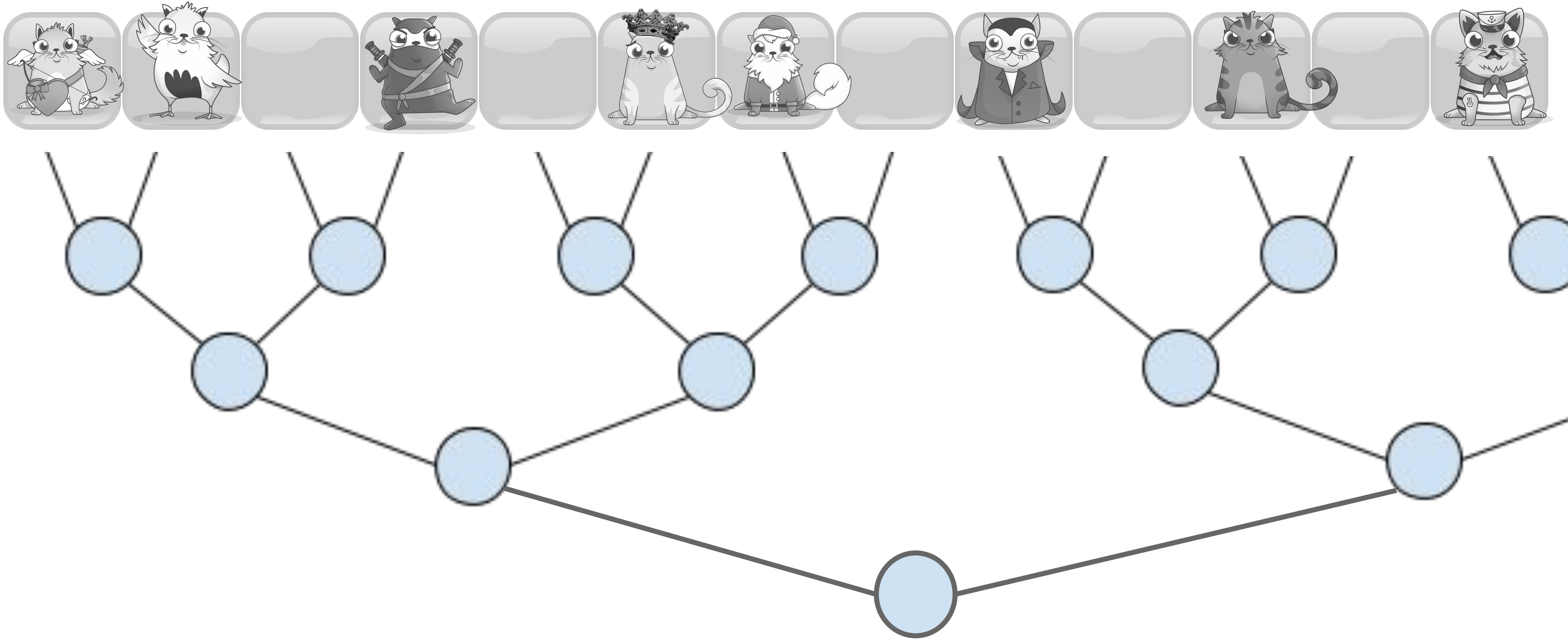


[Dwork et al. 10, CSS'10]

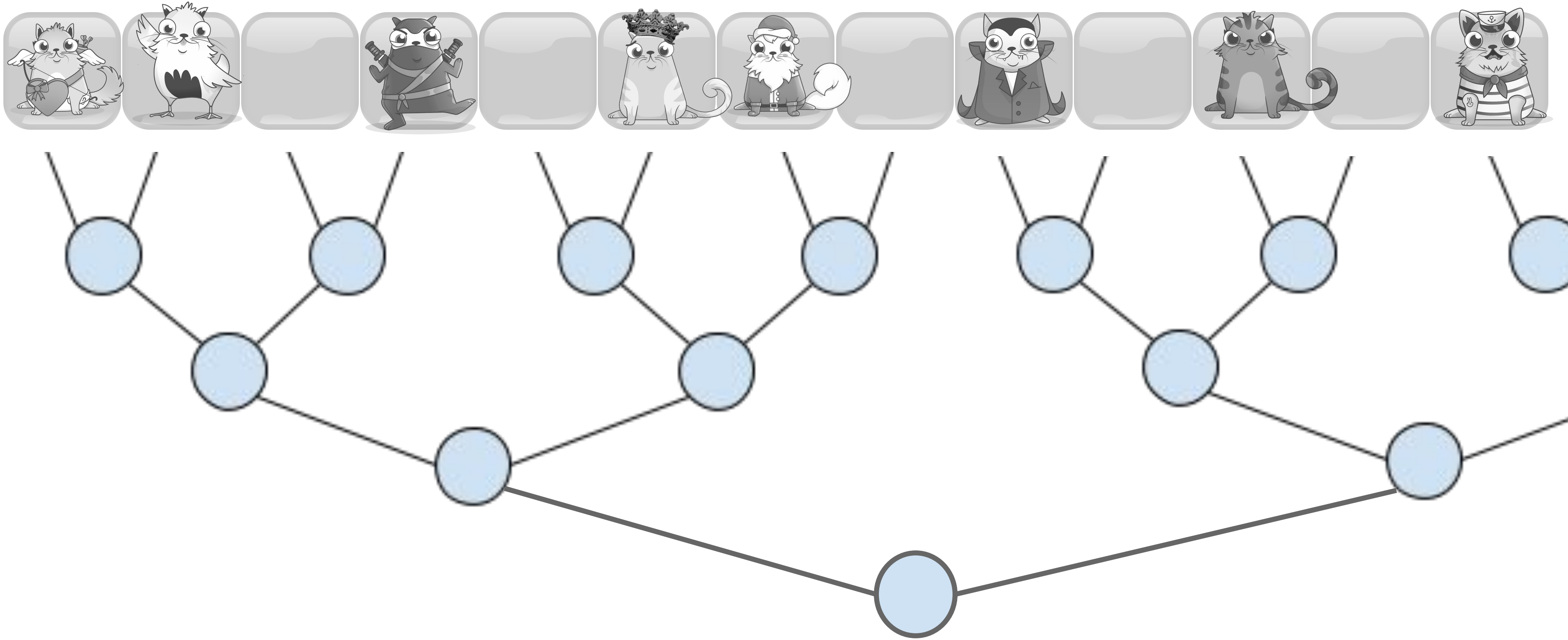
All prefix sums -- DP and Oblivious



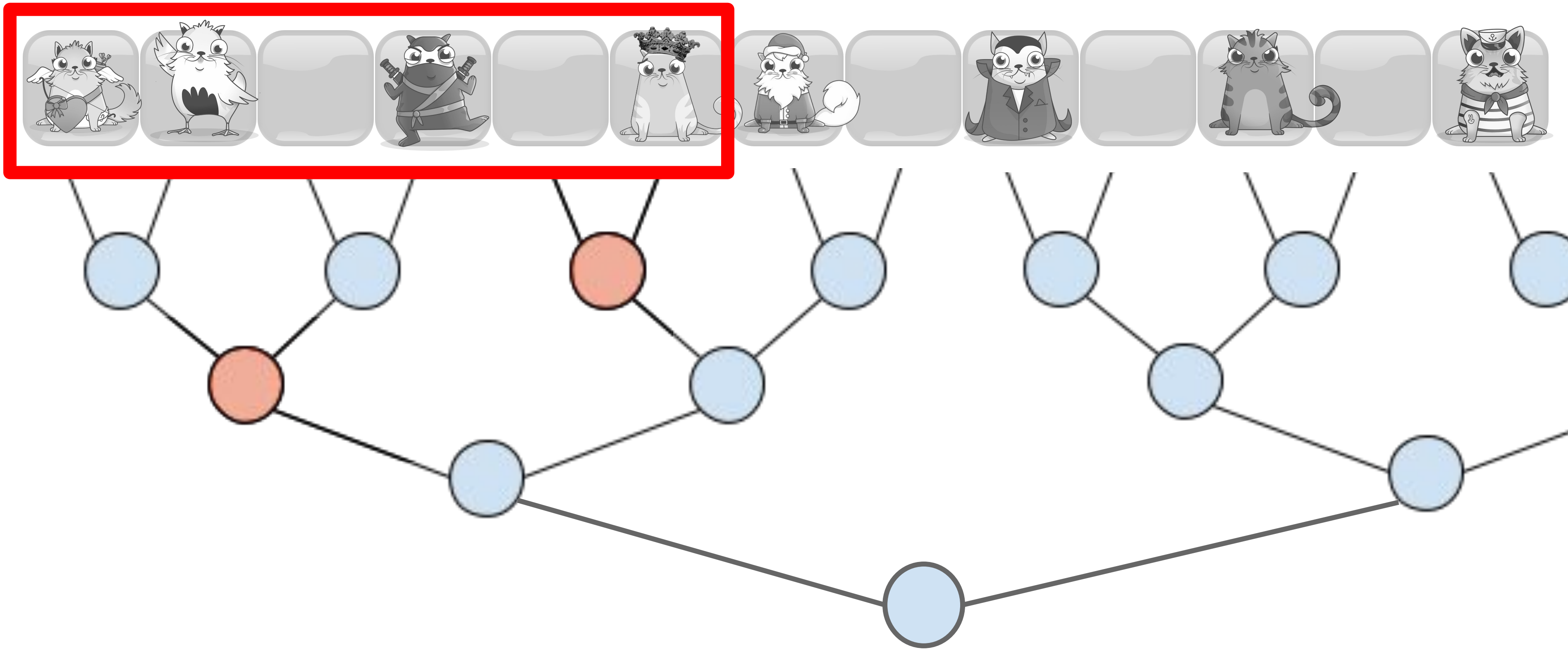
- Every node in the tree represents a range



- Every node in the tree represents a range
- Compute DP estimate for every node in the tree



- Every input appears in only **log N** nodes!
- Achieve only **Θ(log N)** error per node!



- Every prefix sum is the sum of **log N** nodes
- Achieve **poly log N** error for each prefix sum

Summary: Leak rough notion of progress

Non-trivial combination of DP and oblivious algorithms

Apply oblivious alg to small bins

Make DP mechanisms oblivious

Putting it altogether

There exists an $O(N \log \log N)$ time, $(\Theta(1), \mathrm{negl}(N))$-DO algorithm that realizes stable compaction

Is this necessary?

There exists an $O(N \log \log N)$ time, $(\Theta(1), \mathrm{negl}(N))$-DO algorithm that realizes stable compaction

$(\epsilon, 0)$ -Differentially Oblivious Stable Compaction:

N log N is necessary

even when ϵ is arbitrarily large!

Other Results in Our Paper

- **lower bounds for obliviousness**
- **merging, range query DB**
    - ➢ Differentially oblivious algorithms with $O(\log \log N)$ blowup.
    - ➢ $\Omega(\log N)$ blowup necessary for full obliviousness.

Closely Related Works

[Wagh et al.] DP-ORAM, achieve O(1) gain

[Kellaris et al.] DP for length, otherwise fully oblivious

[Mazloom et al.] DP access patterns for MPC

This is just a beginning.

Differential obliviousness for generic programs?

Composition?

Alternative notions?

Practical performance?

This is just a beginning.

Differential obliviousness for generic programs?

Composition?

Alternative notions?

Practical performance?

Thank you!

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